## Structural Depth

## Lateral System - Background Information

Connections in buildings have always been an important issue to consider when going through the design process. The two main types of connections used are fully restrained and partially restrained connections. Fully restrained connections are designed to not allow any rotation at the connection and therefore preventing any moment transfer. A partially restrained connection is a connection that will allow the ends of a beam to rotate slightly to help transfer some of the lateral moment loading. The connection must be designed to flex far enough to allow rotation before the connection fractures.

The graph shown here is and example of End Moments versus Rotation for different types of connections. Curve one represents a fully flexible connection which yields at low moment allowing the connection to rotate. This type of curve is usually attained from top angle or top plate connections. The second curve is the semi-rigid or partially restrained connection. This connection has a varying level of rigidity depending on the type of connection in place and specifically is based off of the slope of the initial stiffness. Connections in this category can include top and bottom angles, top and bottom plates, as well as a combination of the two. Curve 3 represents a fully rigid connection as there is almost no rotation with the introduction of moment. These connections are usually associated with short stiff plates used at the columns.


FIGURE 2

While partially restrained moment connections are not often used en masse in lateral design, some firms such as Stanley D. Lindsey and Associates Ltd. have shown that buildings which utilize PR connections can result in very economical designs. Fabrication designs are not complicated and most welding is eliminated as the connections are simple in design. While this is mostly true for the Hershey Academic Support Center, not all welding is avoided in the use of PR

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connections in the building and with a total of 16 different specifications; the simple design becomes slightly more complex.

To model the partial fixity of a moment connection, there are two defining equations which can be used to find this value. From a paper by John Christopher and Reidar Bjorhovde on Semi-Rigid Frame design, the equations are given as:
where
$\mathrm{R}_{\mathrm{ki}}=$ initial stiffness factor

$$
M=\frac{R_{k i} \phi}{\left[1+\left(\frac{\phi}{\phi_{0}}\right)^{n}\right]^{1 / n}}
$$

$\mathrm{n}=$ shape factor
$\varphi_{o}=$ reference plastic rotation, calculated as $\varphi_{o}=M_{u} / R_{k i}$
$\mathrm{M}_{\mathrm{u}}=$ ultimate moment capacity of the connection
and

$$
\alpha_{i}=\frac{E I}{R_{k i} d}
$$

where
$\mathrm{E}=$ modulus of elasticity
$\mathrm{I}=$ moment of inertia of the beam
$\alpha_{i}=$ non-dimensional characteristic length factor
d = beam depth

These two equations were used to compare the fixity of the different types of moment connections.

Another method of comparison that was used to determine the moments transferred through the partially restrained moment connections is from the Blodgett, Lincoln Arc Welding Foundation as seen on the next page below. Each different type of connection has its own moment equation to describe the behavior of the moment across the end. It is important to note that the connections listed are shown as welds but that the angled connection with bolts performs similarly to one with welded ends, so the values shown are comparable to the connections found in the Hershey Academic Support Center.


## Lateral System - Calculations

The first calculation was to check to make sure the moment connections in the building would yield before fracturing or weld rupturing. If any connections were to fracture or rupture before reaching their yield strength then no moment could be transferred across the connection. Due to the nature of semi-rigid

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connections, it is important that the connections will yield. The equations used were:

Fracture $=\varnothing \mathrm{F}_{\mathrm{u}} \mathrm{A}_{\mathrm{n}}=(0.75) *(58 \mathrm{ksi}) * \mathrm{~A}_{\mathrm{n}}=43.5 \mathrm{ksi} *\left(\mathrm{~A}_{\mathrm{n}}\right)$
Rupture $=\varnothing \mathrm{F}_{\mathrm{n}} \mathrm{A}_{\mathrm{w}}=(0.75) *(0.6)^{*}(70){ }^{*} \mathrm{~A}_{w}=31.5^{*} \mathrm{~A}_{w}$
Yield $=\varnothing$ FyAg $=(0.9) *(36 \mathrm{ksi}) * \mathrm{~A}_{\mathrm{g}}=32.4 \mathrm{ksi}{ }^{*}\left(\mathrm{~A}_{\mathrm{g}}\right)$
where An is the net area of fracture, $\mathrm{A}_{\mathrm{w}}$ is the weld area and Ag is the gross area of the connection.
$\mathrm{MC}-1 \& \mathrm{MC}-2: \mathrm{A}_{\mathrm{n}}=6.48 \mathrm{in}^{2}$ and $\mathrm{Ag}_{\mathrm{g}}=7.98 \mathrm{in}^{2}$
Fracture $=281.88 \mathrm{k}$, Yield $=258.55 \mathrm{k}$, Fracture $>$ Yield ALLOW
MC-3: $\mathrm{A}_{\mathrm{n}}=3.25 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=4 \mathrm{in}^{2}$
Fracture $=150.08 k$, Yield $=129.6 k$, Fracture $>$ Yield ALLOW
MC-4, MC-5, \& MC-7: $\mathrm{A}_{\mathrm{n}}=5.44 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=6.94 \mathrm{in}^{2}$
Fracture $=236.64 \mathrm{k}$, Yield $=224.86 \mathrm{k}$, Fracture $>$ Yield ALLOW
MC-6: $\mathrm{A}_{\mathrm{n}}=3.86 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=4.61 \mathrm{in}^{2}$
Fracture $=167.91 k$, Yield $=149.04 k$, Fracture $>$ Yield ALLOW
MC-8 \& MC-10: $\mathrm{A}_{\mathrm{n}}=2.5 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=3.25 \mathrm{in}^{2}$
Fracture $=108.75 \mathrm{k}$, Yield $=105.3 \mathrm{k}$, Fracture $>$ Yield ALLOW
MC-9: $\mathrm{A}_{\mathrm{n}}=2.88 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=3.63 \mathrm{in}^{2}$
Fracture $=125.28 \mathrm{k}$, Yield $=117.61 \mathrm{k}$, Fracture $>$ Yield ALLOW
MC-11 Top: $\mathrm{A}_{\mathrm{w}}=4 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=2.5 \mathrm{in}^{2}$
Fracture $=126$ k, Yield $=81 \mathrm{k}$, Rupture $>$ Yield ALLOW
MC-11 Bottom: $\mathrm{A}_{\mathrm{w}}=10 \mathrm{in}^{2}$ and $\mathrm{Ag}_{\mathrm{g}}=3 \mathrm{in}^{2}$
Fracture $=315 \mathrm{k}$, Yield $=97.2 \mathrm{k}$, Rupture $>$ Yield ALLOW
MC-12 Top: $\mathrm{A}_{\mathrm{w}}=6 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=3.5 \mathrm{in}^{2}$
Fracture $=189 \mathrm{k}$, Yield $=113.4 \mathrm{k}$, Rupture $>$ Yield ALLOW
MC-12 Bottom: $\mathrm{A}_{\mathrm{w}}=14 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=4.5 \mathrm{in}^{2}$
Fracture $=441 \mathrm{k}$, Yield $=145.8 \mathrm{k}$, Rupture $>$ Yield ALLOW
MC-13 Top: $\mathrm{A}_{\mathrm{w}}=6 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=2.5 \mathrm{in}^{2}$
Fracture $=189 \mathrm{k}$, Yield $=81 \mathrm{k}$, Rupture $>$ Yield ALLOW
MC-13 Bottom: $\mathrm{A}_{\mathrm{w}}=12 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=3.75 \mathrm{in}^{2}$
Fracture $=378 \mathrm{k}$, Yield $=121.5 \mathrm{k}$, Rupture $>$ Yield ALLOW

MC-14 \& MC-16 Top: $\mathrm{A}_{\mathrm{w}}=5 \mathrm{in}^{2}$ and $\mathrm{A}_{\mathrm{g}}=1.5 \mathrm{in}^{2}$
Fracture $=157.5 \mathrm{k}$, Yield $=48.6 \mathrm{k}$, Rupture $>$ Yield ALLOW
$\mathrm{MC}-15$ Top: $\mathrm{A}_{\mathrm{w}}=6 \mathrm{in}^{2}$ and $\mathrm{Ag}=1.88 \mathrm{in}^{2}$
Fracture $=189 \mathrm{k}$, Yield $=60.91 \mathrm{k}$, Rupture $>$ Yield ALLOW
Both sets of calculations passed for all connections so it is safe to assume the connections will transfer moment.

With 617 total connections in the building, some assumptions were made due to the similar nature between beam sizes in the effort to save time. When calculating the individual connection stiffnesses, each angled connection was taken in conjunction with the beam it was most commonly found on and this was assumed to be the average for that connection. The initial stiffness becomes the initial slope for the connection's Moment vs. Rotation graph and can be checked accordingly. For the plates, the stiffness was calculated using a reference graph from W. McGuire on Steel Structures. To test the validity of the graphs with my connections data, the calculated angle connections were compared with the data on the graph using relative area as a basis for comparison. The values came out very similar which can be seen in the graph below.

(a) Riveted connections
(b) Weided connections

Fig. 6.67. Sample $M-\theta$ diagrams.
Steel Structures, W. McGuire, Prentice-Hall 1968.

| Connection Designation | Connection Type | Connection Size | Relative Stiffnesses $\left(R_{k i}\right)$ |
| :---: | :---: | :---: | :---: |
| MC-1 | Top and Bottom Angles | L6 X $4 \times 7 / 8 \times 0{ }^{\prime}-7{ }^{\prime \prime}$ | 101,549 |
| MC-2 | Top and Bottom Angles | L6 $\times 4 \times 7 / 8 \times 0{ }^{\prime}-6{ }^{\prime \prime}$ | 97,589 |
| MC-3 | Top and Bottom Angles | L3-1/2 X 3-1/2 $\times 5 / 8 \times 0$ - $61 / 2^{\prime \prime}$ | 79,203 |
| MC-4 | Top and Bottom Angles | L6 X $4 \times 3 / 4 \times 0{ }^{-7}{ }^{\prime \prime}$ | 87,551 |
| MC-5 | Top and Bottom Angles | L6 X $4 \times 3 / 4 \times 0$ O-8" | 88,380 |
| MC-6 | Top and Bottom Angles | L4 $\times 4 \times 5 / 8 \times 0$ - $10^{\prime \prime}$ | 79,417 |
| MC-7 | Top and Bottom Angles | L6 X $4 \times 3 / 4 \times 0$ O-9" | 92,323 |
| MC-8 | Top and Bottom Angles | L3-1/2 $\times 3-1 / 2 \times 1 / 2 \times 0$ ' $61 / 2^{\prime \prime}$ | 68,596 |
| MC-9 | Top and Bottom Angles | L3-1/2 $\times 3-1 / 2 \times 9 / 16 \times 0$-5" | 68,830 |
| MC-10 | Top and Bottom Angles | L3-1/2 $\times 3-1 / 2 \times 1 / 2 \times 0$-10" | 73,001 |
| MC-11 | Top Plate | PL4 X 5/8 X 1'-2" | 262,300 |
|  | Bottom Plate | PL8 $\times 3 / 8 \times 2$-0 | 241,000 |
|  | Equivalent Stiffness |  | 251,650 |
| MC-12 | Top Plate | PL7 X 1/2 X 1'-8" | 248,100 |
|  | Bottom Plate | PL12 X 3/8 X 2'-10" | 212,700 |
|  | Equivalent Stiffness |  | 230,400 |
| MC-13 | Top Plate | PL8 X 3/8 X 1'-8" | 236,600 |
|  | Bottom Plate | P12 X 5/16 X 2'-8" | 214,000 |
|  | Equivalent Stiffness |  | 225,300 |
| MC-14 | Top Plate | PL4 X 3/8 X 1'-6" | 256,000 |
|  | Bottom Angle | L3-1/2 X 3-1/2 X 1/2 X 0'-6 1/2" | 68,596 |
|  | Equivalent Stiffness |  | 162,298 |
| MC-15 | Top Plate | PL5 X 3/8 X 1'10" | 238,700 |
|  | Bottom Angle | L3-1/2 X 3-1/2 $\times 5 / 8 \times 0$ '-6 1/2" | 79,203 |
|  | Equivalent Stiffness |  | 158,952 |
| MC-16 | Top Plate | PL4 X 3/8 X 1'6" | 256,000 |
|  | Bottom Angle | L3-1/2 $\times 3-1 / 2 \times 1 / 2 \times 0$-10" | 73,001 |
|  | Equivalent Stiffness |  | 164,501 |

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The data above shows that in terms of flexibility, angled connections allow the most rotation for the same amount of moment as the other two connection types. For simplicity sake, connections that had $\mathrm{R}_{\mathrm{ki}}$ values within $5 \%$ are shown as the same curve above, though in reality the curves would be slightly different. Every connection has a unique Moment-rotation curve, but it's interesting to note that at low moments all of these connections behave alike. At about 200"k of moment, the connections branch off depending on their type. As a general rule with angles, the thicker the angle is, the less rotation it allows. Oppositely, plates function in a different manner where that the smaller the plate used, the stiffer it is and the less rotation it allows. For connections with both angles and plates, the two separate values were found and an average was taken to find stiffness over the whole connection.

Using the graph above and the $\mathrm{R}_{\mathrm{ki}}$ values obtained from previous calculations, the restraint value ' $R$ ' can be calculated as a percent of moment transferred for each moment connection. Most partially restrained connections fall between $\mathrm{R}=90 \%$ and $\mathrm{R}=\mathbf{2 0 \%}$ for their restraint value, which proved true with the connections in my building. The highest restraint value was from the top and bottom plate connections at $85 \%$ whereas the lowest value was the top and bottom angles with $23 \%$. The calculated values are shown below.

| Moment Connection | Restraint Value (R) |
| :--- | ---: |
| MC-1 | $34 \%$ |
| MC-2 | $33 \%$ |
| MC-3 | $27 \%$ |
| MC-4 | $30 \%$ |
| MC-5 | $30 \%$ |
| MC-6 | $27 \%$ |
| MC-7 | $31 \%$ |
| MC-8 | $23 \%$ |
| MC-9 | $23 \%$ |
| MC-10 | $25 \%$ |
| MC-11 | $85 \%$ |
| MC-12 | $78 \%$ |
| MC-13 | $76 \%$ |
| MC-14 | $55 \%$ |
| MC-15 | $54 \%$ |
| MC-16 | $56 \%$ |

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With the restraint percentages, a 3D SAP2000 model can be created and used to test the story deflection of the entire structure. While there is no officially set criteria for story deflection, $\mathrm{H} / 400$ will be used to test and see if the structure meets the deflection requirements. SAP2000 models of each individual frame were also created to test and see if any moment connections can be removed and have the structure still meet the deflection requirements, possibly saving time and money. The full lateral model is shown below.


Deflections for the entire structure were calculated and three frames were picked for a typical frame in the East, West, and Center section. Results were:

Deflection Calculation H/400: ((69')*(12in/ft))/400 = 2.07in East Section Frame \#12: Story Drift $=1.53 \mathrm{in}<2.07 \mathrm{in}$ ALLOW West Section Frame \#2: Story Drift $=1.47 \mathrm{in}<2.07$ in ALLOW Center Section Frame \#D: Story Drift $=1.87$ in $<2.07$ in ALLOW

All sections passed with the partial fixity in place which shows a good design. This data also shows that partially restrained connections allow more deflection than fully restrained connections when compared with my initial fully restrained data. The original data only analyzed one frame at a time whereas the new data was taken with the entire lateral system supporting itself and yet the deflections were very similar.

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Since each section passed, all three moment frames had two connections removed from the roof section to see if deflection would still pass if fewer connections were in place. The connections from the roof were the chose to be removed because they represent the smallest angles and plates involved in the lateral system due to the braced frames supporting the other direction. Two connections were removed instead of just one to keep the frame symmetric and $t$ he wind loads balanced. The East Section is shown below with the Center and West Sections summarized as well.


The new results after removing two moment connections:

Deflection Calculation H/400: $\left(\left(69^{\prime}\right)^{*}(12 \mathrm{in} / \mathrm{ft})\right) / 400=2.07 \mathrm{in}$ East Section Frame \#12: Story Drift $=1.85 \mathrm{in}<2.07$ in ALLOW West Section Frame \#2: Story Drift $=1.76 \mathrm{in} \leqslant 2.07 \mathrm{in}$ ALLOW Center Section Frame \#D: Story Drift $=2.23$ in $<2.07$ in FAIL

Upon removal of two moment connections, the story drift increased in all three sections with the Center section going over the allotted H/400 level. The

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next test was to remove all the roof connections in the East and West Section to see if the deflection checks would still pass. The West Section is shown below:


The new results after removing the roof moment connections for East \& West:

Deflection Calculation H/400: $\left(\left(69^{\prime}\right)^{*}(12 \mathrm{in} / \mathrm{ft})\right) / 400=2.07 \mathrm{in}$
East Section Frame \#12: Story Drift $=2.28 \mathrm{in} \leqslant 2.07$ in FAIL
West Section Frame \#2: Story Drift $=2.19$ in $<2.07$ in FAIL

Removing all the connections was too much as the deflection of the side sections didn't meet the $\mathrm{H} / 400$ requirement. One last trial was conducted where the moment connections were removed from every other frame on both the East and West Section with the Center section left as designed. The Center section is shown below as well as all three results.


The final results after removing every other frame:

Deflection Calculation H/400: ((69')*(12in/ft))/400 = 2.07in East Section Frame \#12: Story Drift $=2.03 \mathrm{in}<2.07$ in ALLOW West Section Frame \#2: Story Drift $=1.94 \mathrm{in}<2.07 \mathrm{in}$ ALLOW Center Section Frame \#D: Story Drift $=1.96 \mathrm{in}<2.07 \mathrm{in}$ ALLOW

The new system passes the deflection check showing that it is possible to remove some of the smaller moment connections and still have the system work. The total savings of removing 24 total moment connections is valued at approximately $\$ 4,000$ using cost data from the Milton Steel Corporation.

## Floor System - Background Information

The Hershey Academic Support Center uses a special type of floor design known as "Type 2 with Wind". The basic principal for Type 2 with Wind design is to take the negative moment value from the wind force and use this when designing the lateral force member. Members located within the moment frames have a laterally based design while interior beams use the standard gravity load design to choose member sizes. This method ensures that the lateral force will be adequately resisted within the structure, but can often result in varied member types throughout the building. Another factor attributed from Type 2 with Wind design is that shear studs are used to help adjust the balance between the positive moment in the center of a normal gravity load distribution and the negative moment located at the ends. This creates an issue where economy must be considered to pick a member that has an optimum girder size to shear stud ratio. Since the lateral system now uses the partially restrained connections, new moments needed to be calculated and the floor members checked.

## Floor System - Calculations

To assist with the design of new floor members, a RAM Steel Model was created for each floor to see if the new moments would affect the member design. An example floor section from the East Wing is shown under the old moment system:


The new loading data from SAP2000 was entered into RAM and the new floor plans were compared to the old. Most members stayed the same as before but a few changes were noted as shown below:

The North-South spanning members changed from a W21x44 with 17 shear studs to a W18x40 with 16 shear studs on the first and second floors and changed from a W18x40 with 16 shear studs to a W18x35 with 12 shear studs on the third and forth

floors. The top floor experienced no change in member sizes which is most likely due to the removal of some of the moment connections. Both East and West sections experienced this change with a total of 40 W21x44s changing into $\mathrm{W} 18 \times 40$ and $40 \mathrm{~W} 18 \times 4$ os changing into $18 \times 35 \mathrm{~s}$. This totaled up to 6.2 tons of steel between all the members and RMS estimates steel prices at about $\$ 2,000$ per ton of steel, so the total savings was approximately $\$ 12,320$.

